Mathematical Methods for Physics using Maple

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One mathematical technique that is becoming increasingly important is the use of computers to perform *algebraic* calculations. This course will present some of the main mathematical techniques required in theoretical physics and explain how to apply them using the mathematical computation system Maple, which provides algebraic, symbolic, numerical and graphical facilities. It will also include some physical applications. It will therefore not be possible to give as much mathematical rigour or background as would be usual in a course on mathematical methods and instead there will be a significant practical component – students will use Maple themselves to solve problems, guided by the examples discussed during the lectures. After an initial general introduction, Maple facilities will be introduced in detail as they are required to implement the mathematics.

The mathematics will be based mainly on "Methods of Mathematical Physics" by Jeffreys and Jeffreys (Cambridge, 1956) and "Mathematical Methods of Physics" by Mathews and Walker (Benjamin, 1965), but some specialized topics (such as differential geometry and distribution theory) will necessarily also require other sources.

Some experience with computers and with programming in a language such as Pascal or C will be an advantage, but is not essential.

Proposed Syllabus

1. Introduction
    - About the course
    - Worked examples from elementary mechanics
    - About Maple: its use and language syntax

2. Real Analysis

   - Sequences and series; convergence
   - Limits of functions; orders of magnitude
   - Differentiation and Integration: Riemann, Stieltjes, Lebesgue; measure; infinite and improper integrals
   - Functions of many variables
   - Uniform convergence; continuity and integrability
   - Convergence tests
   - Taylor series
   - Infinite products
   - Lipschitz condition, Cauchy-type inequalities

3. Matrices and Linear Algebra

   - Matrix algebra; special properties of matrices
   - Linear equations; determinants
   - Orthogonal transformations
   - Rank; homogeneous linear equations
   - Eigenvalues; diagonalization
   - Block matrices; reduction to special forms
   - Quadratic and Hermitean forms
   - Root separation
   - Rayleigh's principle of stationary eigenvalues
   - Physical applications: small oscillations; Pauli spin and Dirac matrices

4. Scalars and Vectors

   - Cartesian coordinates; transformations
   - Vector algebra; products
   - Tensorial notation, δ and ϵ
   - Physical examples: particle motion; rotation
   - Cartesian tensors

- Vector calculus: grad, div, curl
- Physical example: Maxwell's equations

5. General Coordinate Systems and Multiple Integrals
    - Oblique axes; contra- and co-variant components
    - Physical example: crystal structure
    - Curvilinear coordinates; metrics
    - Integral equations (?)
    - Change of integration variable; Jacobian
    - Line and surface integrals
    - Stokes' and Gauss' Theorems

6. Differential Geometry
    - Space curves and surfaces
    - Curvature and torsion
    - The Frenet formulae
    - The Riemann tensor
    - Physical application: relativity theory

7. Operational and Numerical Methods
    - Differential and integral operators
    - Sets of coupled ODEs
    - Physical applications: LC electric circuits; radioactive decay
    - Polynomial interpolation
    - Numerical solution of equations
    - Integration: Euler-Maclaurin; Newton-Cotes
    - Differential equations
    - Eigenvalues

8. Calculus of Variations
    - Fixed and variable endpoints; Euler-Lagrange equations
    - Several dependent variables

- Physical applications: Fermat's principle; brachistochrone; catenary; Hamilton's principle; Lagrange's equations

9. Complex Analysis
   - Functions of a complex variable
   - Integration; Cauchy's theorem
   - Laurent series; singularities; analytic continuation
   - Contour integration
   - Bernoulli numbers and polynomials
   - Bromwich's contour integral
   - Special contour integrals

10. Fourier Series and Transform Theory
    - Fourier series
    - Integration and differentiation of Fourier series
    - Application to differential equations
    - Gibb's phenomenon
    - Weierstrass' approximation theorem
    - Parseval's theorem
    - Fourier integral transforms; Laplace and Mellin transforms
    - Heaviside step and Dirac delta functions
    - Physical applications: oscillation; frequency response

11. Special Integrals, Orthogonality and Distributions
    - Gamma and beta functions; multigamma (Ψ) functions
    - Wallis' formula for π
    - Exponential, logarithmic and Fresnel integrals
    - Physical application: wave diffraction from an edge
    - Orthogonal functions and polynomials
    - Distribution theory

12. Ordinary Differential Equations
    - Power series solutions; singular points
    - Recurrence relations and difference equations
    - Integral representations of solutions
    - Wronskians; variation of parameters
13. Asymptotic Expansions
    - Their nature and definition
    - Steepest descents and stationary phase
    - Stirling's formula
    - The Airy integral function
    - Wave dispersion phenomena
    - Asymptotic solutions of ODEs
14. Partial Differential Equations
    - Types: elliptic, parabolic and hyperbolic
    - Physical origins: Laplace, diffusion and wave equations
    - Separable solutions; solution by integral transforms
15. Hypergeometric-type Special Functions
    - Bessel functions
    - Physical applications: cylindrical waves
    - Legendre and associated functions; spherical harmonics
    - Physical applications: spherical waves
    - Hypergeometric functions
    - Elliptic functions